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Abstract

The difficulty encountered in designing an interaction region (IR) for a 2-TeV Muon Collider lies in the extreme constraints placed on beam parameters at the point of collision. This paper examines a relaxation of the interaction-point criterion insofar as it impacts luminosity, the design, and stability of the interaction region.

I. INTRODUCTION

It has been suggested for the muon collider that the transverse beam parameters at the interaction point (IP) have the values $\beta^*_{x,y}$ =3 mm (rms) for a beam with a normalized emittance of $\epsilon_N = 50\pi$ mm-mrad [1]. These stringent requirements were imposed in order to overcome partially the hour-glass effect and retain a luminosity of $\mathcal{L} = 3 \times 10^{33}$ cm^{-2} s^{-1} in the collision process; that is, assuming two 3 ps rms bunches of opposite charges populated by $N = 2 \times 10^{12}$ muons each. An additional requirement is that a ± 150 mrad detector acceptance must be maintained about the collision point. This implies 2-6 m of separation must be allowed between the first IR quadrupole and the interaction point. The 3-mm low-beta and a long interaction drift produce a very sensitive and highly achromatic interaction region. The design is further hindered by the reduced gradient strengths of quadrupoles at 2-TeV.

Because the betatron amplitude scales as the square of the distance from the IP divided by β^*, the most direct approach to reduce chromaticities in the IR optics is to increase β^*. Moving quadrupoles closer to the interaction point in principle should also work, but in practice, unusually weak bending power of the quadrupoles reduce greatly any impact on the betatron functions. The purpose of this paper is to assess to what degree luminosity is affected by an increase in β^* and what gains are made insofar as the IR is concerned.

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II. LUMINOSITY CONSIDERATIONS

The interaction region of a muon collider is embedded in a storage ring, yet it must exhibit the tight final focus properties of a linear collider in order to achieve the projected luminosity. This represents the most critical issue in the design of the 2-TeV collider ring.

The actual or effective luminosity $\mathcal{L}_{\rm eff}$ achievable in a 2-TeV muon collider ring is modified from the nominal luminosity, $\mathcal{L}_0$ which is given by the standard equation:

$$\mathcal{L}_0 = \frac{N^2 \gamma f}{4\beta^* \epsilon_N}\,, \tag{2.1}$$

where γ is the energy of the beam divided by the muon mass and f is the frequency of collision. At high muon intensities beam-beam interactions become important and enhance the effective luminosity due to the mutual focusing effect one colliding beam has on the other. A disruption parameter can be defined which characterizes the degree of beam-beam focusing. The other contribution that must be considered, the so-called hour-glass effect, arises from the rapidly increasing betatron functions, or transverse beam sizes, away from the collision point. The hour-glass effect is a measure of the depth of field and is strictly geometrical; it reduces the overall luminosity because of the dilution in particle density, and counteracts the disruption enhancement.

A prototype IR with a low-beta of 3 mm had been designed by Gallardo and Palmer [2]. Recently, Chen [3] performed a numerical simulation for the effective luminosity which included both the disruption and geometrical effects and obtained a luminosity enhancement factor of 0.87. Since the geometrical, or hour-glass, effect has been calculated independently to be 0.76, this implies that disruption causes a 14% increase in luminosity.

Relaxing the low-beta constraint for collision impacts the luminosity, but it facilitates achieving a workable IR design. The impact of β^* on luminosity can be estimated by noting that as β^* increases both the disruption-focusing and hour-glass effects disappear rapidly. The enhancement factor becomes one and effective luminosity, $\mathcal{L}_{\rm eff}$, approaches the nominal, $\mathcal{L}_0$. According to Eq. (2.1), the effective luminosity then scales inversely to the ratio of β^*. With the enhancement factor at 3 mm taken into account, modified initial luminosity factors for β^*'s of 1 and 3 cm are given in the middle column of Table I.

The Gallardo-Palmer IR has a long length of about 947 m. The reason for this is that its high natural chromaticities of ~ -6000 have to be corrected locally. Either side of the IP is flanked by four high-beta, high-dispersion bumps. As will be shown below, IR's with $\beta^* = 3$ cm and 1 cm have chromaticities of only ~ -270 and ~ -500, respectively, making local correction at the IR unnecessary. As a result, their length can as short as ~ 200 m. For a collider ring of radius ~ 1 km with two IR's, this represents a 24% increase in revolution or collision rate and therefore luminosity. Taking into account path-length effects, the effective luminosity is recomputed and tabulated in the last column of Table I.

Disruption focusing also dilutes the emittances in both planes lowering the luminosity. Preliminary calculations by Chen [3] give a fractional increase of $\delta\epsilon_N/\epsilon_N = 6\%$ for the first interaction, but it is much less in subsequent collisions. Including emittance dilution may further reduce the discrepancy between 3 mm and centimeter IRs.

From the above analysis it is clear that the 3 mm β^* criteria could be relaxed to 1 or 3 cm without a severe sacrifice in luminosity.

Table I: Luminosity reduction versus β^*.

β^*	Reduction Ratio	
(cm)	H & D	Total
0.3	1.0	1.0
1.0	2.9	2.3
3.0	8.7	7.0

III. LOW-BETAS AND CHROMATICITIES

A relaxed β^* contributes enormously to the stability of the muon collider; in particular with respect to chromaticity, which has been a persistent problem in designing a 3 mm, 2-TeV IR. This can be seen from the simplified IR depicted in Fig. 1, where there are only two *thin* quadrupoles QF and QD to focus the beam horizontally and vertically, respectively.

The maximum of the vertical betatron function occurs at QD and is approximately equal to

$$\beta_y \gtrsim \frac{s_D^2}{\beta^*}\,, \tag{3.1}$$

where s_D is the distance measured from the collision point. The small beam size at the focus, or collision point, in the IR causes transverse sizes to increase rapidly as a function of distance from the IP. In order to reverse the rise of the β functions, the focal length of IR quadrupoles must be

$$f \lesssim s_F\,, \tag{3.2}$$

where s_F is the distance of the center of QF from the collision point. The vertical chromaticity is given very roughly by

$$\xi_y \sim -\frac{1}{4\pi}\frac{\beta_y}{f} \sim -\frac{s_D^2}{4\pi s_F \beta^*}\,. \tag{3.3}$$

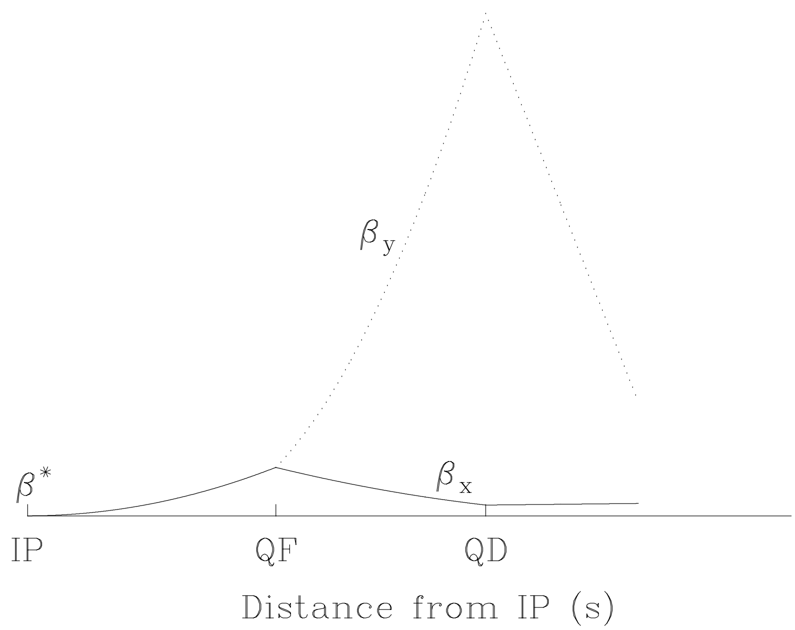


Figure 1: A quadrupole doublet scheme at the IR.

Because of the high energy of this ring, the superconducting quadrupoles encounter gradient limitations, particularly in IR region. A pole-tip field of 9.5 T yields a quadrupole strength of only $k \lesssim .04\ \mathrm{m}^{-2}$ at $\beta = 15$ km, or greater. Such weak gradients result in long quadrupoles, causing the amplitude in the defocusing direction to skyrocket inversely proportional to the value of β^*. (see Eq. (3.1)). For example, the quadrupole length is given by $\ell = 1/(fk)$. Therefore, although the distance of the interaction point to the first quadrupole is 6.5 m, the distance to the center of the first quadrupole s_F is 9 to 10 m, and s_D can be 20 to 40 m. The resulting chromaticity, as given by Eq. (3.3), is naturally large because of the distances and quadrupole lengths involved. In any case, very roughly, the natural chromaticities scale inversely with β^*.

Knowing the natural chromaticities of the Gallardo-Palmer IR, chromaticities of IR's with larger β^* can be estimated by scaling to the $\beta^* = 3$ mm case:

$$\begin{array}{ll} \beta^* = 3\ \mathrm{mm}, & \xi_y \sim -6000 \\ \beta^* = 1\ \mathrm{cm}, & \xi_y \sim -1800 \\ \beta^* = 3\ \mathrm{cm}, & \xi_y \sim \ -600 \end{array}$$

One observes from these numbers that an order of magnitude improvement can be made in the chromaticity paying an equivalent price in β^*. Furthermore, scaling has overstimated actual values. This is because with larger β^*'s, transverse beam sizes in the IR will be much smaller. Smaller beam apertures mean quadrupoles with higher gradients can be used and they will be more effective in controlling the betatron functions. Table II displays the minimum aperture required as a the function of betatron amplitude for a 5-sigma beam width plus about 5 mm for shielding. Alongside these data are the maximum field gradient, B', and the strength, $k = B'/(B\rho)$, assuming a pole-tip field of 9.5 T. In Fig. 2, we plot dynamical parameters for a 5σ beam aperture and a beam energy of 2-TeV.

Table II: Maximum quadrupole gradient as a function of β for a 2-TeV collider.

β (km)	Aperture (cm)	Gradient (T/m)	k (m^{-2})
1	0.81	1170	0.175
3	0.41	675	0.101
5	1.82	522	0.078
7	2.15	442	0.066
9	2.44	390	0.058
10 – 15	$\sim$ 3.0	$\sim$ 325	0.049
15 – 20	$\sim$ 3.3	$\sim$ 375	0.043
20 – 25	$\sim$ 3.8	$\sim$ 245	0.037
25 – 30	$\sim$ 4.2	$\sim$ 220	0.034
30 – 35	$\sim$ 4.6	$\sim$ 205	0.030
35 – 40	$\sim$ 5.0	$\sim$ 190	0.028
40 – 50	$\sim$ 5.4	$\sim$ 175	0.026
50 – 70	$\sim$ 6.2	$\sim$ 150	0.023
70 – 90	$\sim$ 7.4	$\sim$ 130	0.020
90 – 110	$\sim$ 8.1	$\sim$ 115	0.017

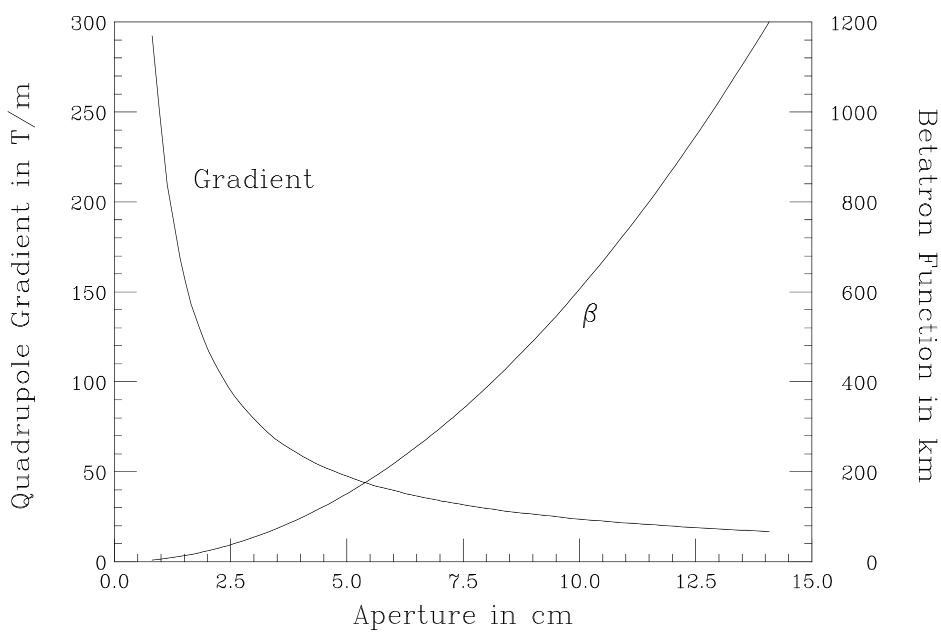


Figure 2. Dynamical parameters of collider at 2 TeV.

Here, we would like to point out that a $\mu^+\mu^-$ collider is very much different from an e^+e^- collider. Muons are decay products of pions which are in turn secondary particles produced by protons hitting a target, and cannot be cooled efficiently due to the short muon life time. As a result, the emittances for muons are, in general, much larger than those for the electrons. The transverse amplitudes of a muon bunch will also be much larger than for electron bunches. For this reason, the allowable quadrupole gradients will be more restricted in a muon collider than an e^+e^- collider.

IV. A 3-CM INTERACTION REGION

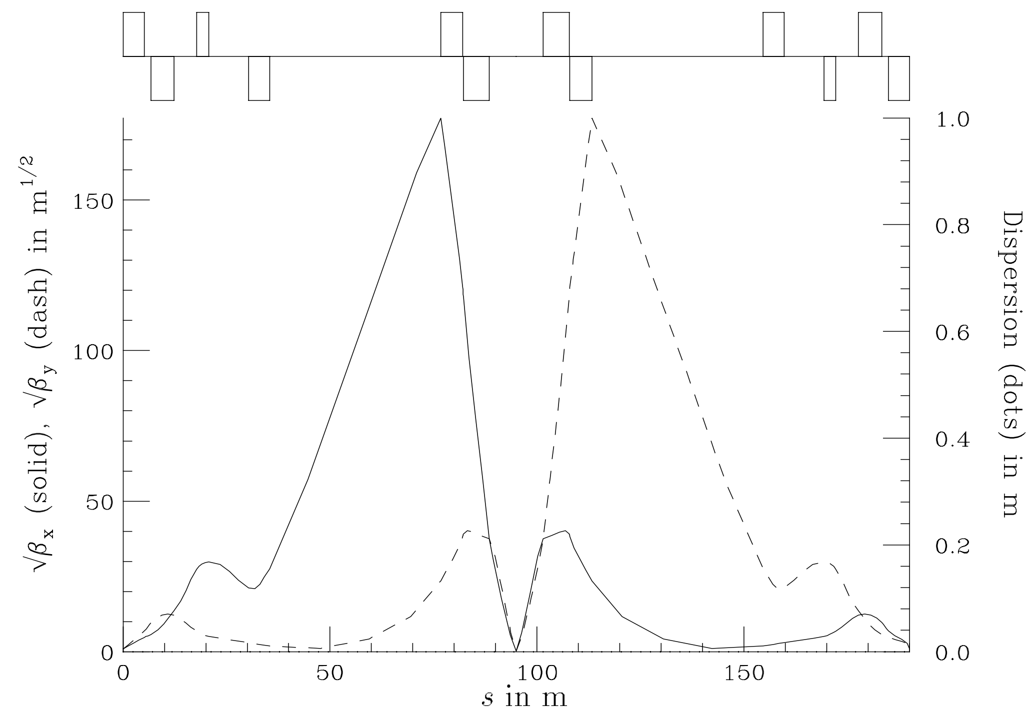
A. The IR Lattice

A stable, realistic IR with a 3-cm low-beta has been designed. It is comprised of essentially two quadrupoles; i.e., a doublet, as indicated in Fig. 1. Here, because of the much slower rise of the betatron functions as a function of distance from the point of collision, the first quadrupole gradient is strong enough to focus effectively before entering the second quadrupole. Consequently, a standard focusing doublet can be employed so that the β functions in both planes reach a minimum in approximately 50 m. In this design the first quadrupole is placed at the nominal 6.5 m from the IP. The maximum betatron function, as can be seen from the figure is 31.3 km, which is to be compared with the 400-km peak β in the 3 mm, Gallardo-Palmer IR. The 1-cm IR displayed in Figure 3 is an antisymmetric arrangement which allows the natural chromaticities to be minimized in both planes. The natural chromaticities have been optimized to values of $\xi_x = \xi_y = -270$.

Also investigated was the possibility of positioning the quadrupoles closer to the IP. We reduced the 6.5 m separation to 3 m. With lengths of approximately 6.5 m, the centers of the first and second quadrupoles are $s_F = 9.75$ m and $s_D = 16.25$ m away from the IP. Reducing the distance to the IP by 3 m shortens them to 6.25 and 12.75 m, respectively. Therefore, according to Eq. (3.3), we expect the chromaticity to decrease by approximately a factor of 1.04. In actual design, a reduction factor of 1.2 was found. It can be concluded that moving the quadrupoles nearer to the IP does not result in a large reduction in the chromaticity of the region.

In order to obtain some insight into the problem, we need to introduce the *bending power* or *bending efficiency* of the first IR quadrupole [4], for its effect of altering the slope of the betatron function. The change in Twiss parameter α across the quadrupole is given by

$$\Delta\alpha = \frac{\beta}{f}\,. \tag{4.1}$$



Dispersion max/min: 0.00000/−0.00000m, γ_t: (0.00, 0.00)
β_x max/min: 31433.13/ 0.03000m, ν_x: 1.56763, ξ_x : −270.32, Module length: 190.0354m
β_y max/min: 31433.13/ 0.03000m, ν_y: 1.58763, ξ_y : −270.32, Total bend angle: 0.00000 rad

Figure 3. The antisymmetric 3-cm β^* IR.

The maximum strength of the quadrupole depends on the transverse size of the beam, and therefore the betatron function; thus, $f \propto \sqrt{\beta}$. Therefore, the bending power of the first IR quadrupole is

$$\Delta\alpha \propto \sqrt{\beta} \propto s_F \, . \tag{4.2}$$

As a result, when s_F is reduced, although the quadrupole strength is increased, its ability to bend the betatron function has, in fact, been lowered. This explains why the chromaticity will not decrease in the same way s_F decreases.

Since the outside of a superconducting quadrupole can be contained within a 40 cm radius, the clearance angle at the IP is $\theta \lesssim \pm\, 61$ mr when the IP-quadrupole separation is 6.5 m and $\lesssim \pm\, 134$ mr when it is 3.0 m. These angles are well below the detector acceptance criterion of $\theta \leq \pm 150$ mr.

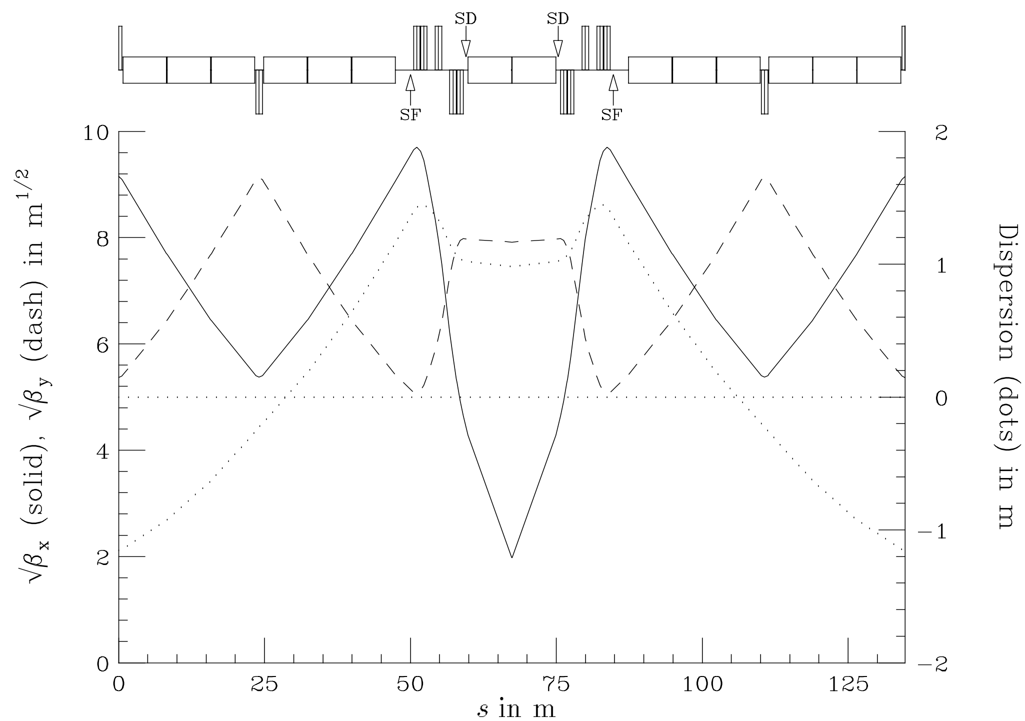
B. CHROMATICITY CANCELLATION

Because the inherent natural chromaticities of this IR are so small, local correction at the IR can be avoided. Sextupoles can be located in the arcs to do this job, reducing the length of the IR tremendously.

In lieu of a more complete design for the entire collider ring, we have used a quasi-isochronous $\beta^* = 3$ mm ring design from earlier work for computational purposes. Because of huge chromaticities arising from two $\beta^* = 3$ mm IR's, cancellation of chromaticities using only sextupoles in the arcs was never successful. In this work, we replace the IR's with 3-cm IR's and proceed to estimate the strengths of the correction sextupoles required. The constructed ring is two-fold symmetric. Half of it is composed of

$$\text{Half Ring} = 21 \text{ Normal Modules} + \text{IR} + \text{Zero-Dispersion Straight} ,$$

where the *normal module* is a flexible momentum-compaction module [5] depicted in Fig. 4. With the previous lattice and this new IR, chromaticities were obtained which are listed in Table III.



Dispersion max/min: 1.44677/−1.15250m, γ_t: (1066.87, 0.00)
β_x max/min: 94.11/ 3.90830m, ν_x: 0.75000, ξ_x : −1.32, Module length: 134.7766m
β_y max/min: 83.55/25.71391m, ν_y: 0.46953, ξ_y : −0.65, Total bend angle: 0.14096690 rad

Figure 4. The normal module.

For the correction scheme, two horizontal and two vertical sextupoles SF and SD can be placed in each basic module. In practice, sextupoles were placed only in 20 modules. This is because each module has a horizontal tune advance close to 270° and the nonlinearity due to sextupoles will be roughly canceled

Table III: Chromaticity for half a 3 cm β^* storage ring.

	IR	Arc	Total
ξ_x	-270	-176	-446
ξ_y	-270	-44	-314

every 4 consecutive modules. Using SYNCH [6], the integrated sextupole strengths required to reduced ξ_x by -446 and ξ_y by -314 were computed. The results are $S_F = 1.405$ m^{-2} and $S_D = -2.298$ m^{-2}.

If one assumes a maximum pole-tip field of 9.5 T for these sextupoles with a half aperture of 3 cm, we obtain $B'' = 2111$ T/m^2. For a muon energy of 2 TeV, the sextupole lengths are calculated to be 0.44 m for the horizontal and 0.73 m for the vertical. The space available in the normal module design is 3 m for SF and 0.82 m for SD. The betatron functions and dispersion at these points are listed in Table IV. Obviously, further optimization can be done; for example, increasing β_y and dispersion will shorten the length of SD.

Table IV: Lattice functions at sextupoles.

	β_x	β_y	D
S_F	95.3 m	26.0 m	1.295 m
S_D	18.5 m	64.2 m	1.038 m

C. NONLINEAR TUNE SPREADS

Chromaticity-correction sextupoles give rise to nonlinearities in the lattice optics. The most important nonlinear terms to evaluate are the second-order tune spreads as a function of transverse amplitudes. These are given by

$$\begin{aligned}\delta\nu_x &= \alpha_{xx}\frac{\epsilon_x}{\pi} + \alpha_{xy}\frac{\epsilon_y}{\pi}\,, \\ \delta\nu_y &= \alpha_{yy}\frac{\epsilon_y}{\pi} + \alpha_{xy}\frac{\epsilon_x}{\pi}\,, \end{aligned} \tag{4.3}$$

where ϵ_x and ϵ_y are the unnormalized transverse emittances of the muon beams. Calculations using SYNCH give for this arrangement of sextupoles for the *half* collider ring,

$$\begin{aligned}\alpha_{xx} &= -0.263\times 10^6\ \mathrm{m}^{-1}\,, \\ \alpha_{xy} &= -0.566\times 10^5\ \mathrm{m}^{-1}\,, \\ \alpha_{yy} &= +0.492\times 10^5\ \mathrm{m}^{-1}\,. \end{aligned} \tag{4.4}$$

Using 2σ for a transverse beam size, then $\epsilon_x = \epsilon_y = 1.056 \times 10^{-8}\pi$ m, and the maximum tune spreads are

$$\Delta\nu_x = -0.0034 \quad \text{and} \quad \Delta\nu_y = -0.0006\,, \tag{4.5}$$

which are clearly stable. For comparison, the Gallardo-Palmer [2] tune shift coefficients are

$$\begin{aligned} \alpha_{xx} &= -0.564 \times 10^{9}\ \mathrm{m}^{-1}\,. \\ \alpha_{xy} &= -0.931 \times 10^{8}\ \mathrm{m}^{-1}\,, \\ \alpha_{yy} &= -0.315 \times 10^{10}\ \mathrm{m}^{-1}\,, \end{aligned} \tag{4.6}$$

leading to large tune spreads for a single IR of:

$$\Delta\nu_x = -6.9 \quad \text{and} \quad \Delta\nu_y = -34\,. \tag{4.7}$$

We do learn that Gallardo and Palmer have recently improved the sextupole placements in their IR and brought the tune spreads down to an acceptable value of ~ 0.08. However, the correction scheme at the IR remains very complicated.

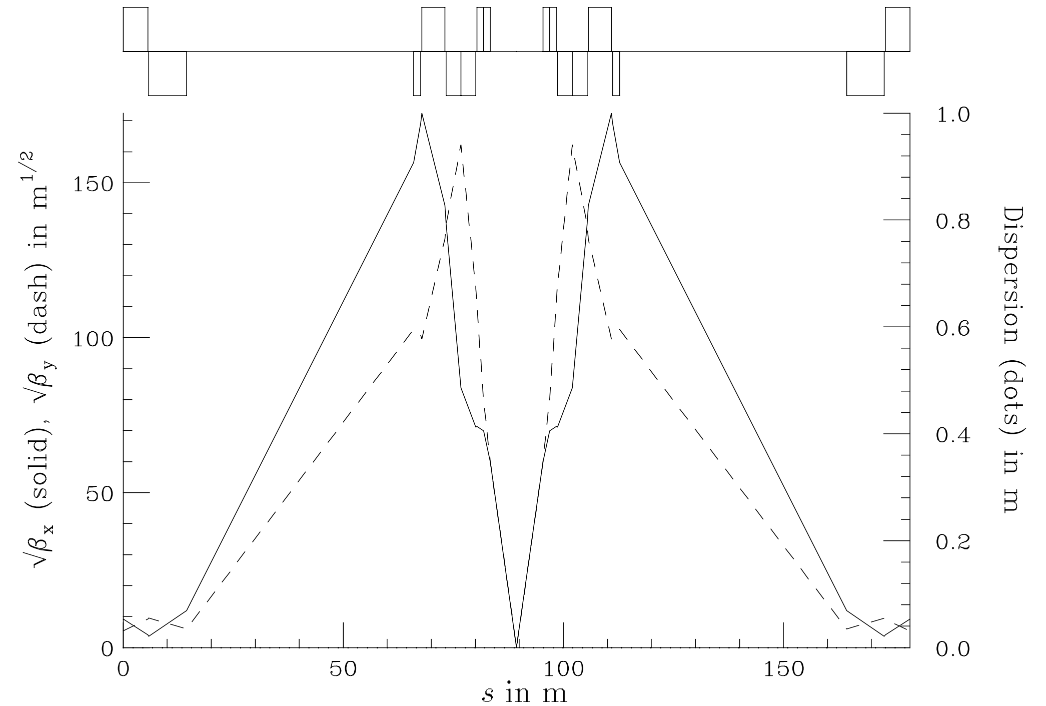
V. THE 1-CM INTERACTION REGION

A second IR region has also been attempted producing a β^* of 1 cm with a pair of doublets. Because of the larger horizontal betatron function and, therefore, larger aperture, the first quadrupole has a relatively weaker gradient than the one used in the 3-cm IR. It is just strong enough to damp the rise of β_x before entering the defocusing quadrupole. The relative strengths of the doublets have been adjusted to equalize the maximum betatron amplitudes in both planes. (They attain a maximum of approximately 28 km.) Equal maxima optimize the chromatic properties of the region and allow the IR to be symmetric about the collision point. Again, this is essentially a doublet scheme and provides the minimal chromaticity for a 1 cm low-beta region. In our design, the horizontal and vertical chromaticities are each about -500, independent of whether a symmetric or antisymmetric arrangement is constructed. A symmetric arrangement is shown in Fig. 5.

If we try to correct for the chromaticities as was done for the 3-cm IR, the corresponding sextupole strengths are $S_F = 2.187\ \mathrm{m}^{-2}$ and $S_D = -3.890\ \mathrm{m}^{-2}$ (implying sextupole lengths of 0.68 m and 1.24 m, respectively). Although the sextupole SD is too long to fit into the present normal module, small modifications will provide the required space. A previous design which eliminates one center dipole could be used to supply the extra space needed for sextupoles.

For the 1 cm IR, the nonlinear tune-spread coefficients are

$$\begin{aligned} \alpha_{xx} &= -0.600 \times 10^{6}\ \mathrm{m}^{-1}\,, \\ \alpha_{xy} &= -0.170 \times 10^{6}\ \mathrm{m}^{-1}\,, \\ \alpha_{yy} &= +0.115 \times 10^{6}\ \mathrm{m}^{-1}\,. \end{aligned} \tag{5.1}$$



Dispersion max/min: 0.00000/−0.00000m, γ_t: (0.00, 0.00)
β_x max/min: 29739.78/ 0.01000m, ν_x: 1.45272, ξ_x : −501.21, Module length: 178.7508m
β_y max/min: 26296.47/ 0.01000m, ν_y: 1.54576, ξ_y : −544.96, Total bend angle: 0.00000 rad

Figure 5. The symmetric 1-cm β^* IR.

The maximum tune spreads at a 2σ amplitude in half of the collider ring are therefore

$$\Delta\nu_x = -0.0081 \quad \text{and} \quad \Delta\nu_y = 0.0018\,, \tag{5.2}$$

which are still very small.

VI. CONCLUSION

In conclusion, we have analyzed both the advantages and the disadvantages of raising the low-beta specification for the IR of the muon collider ring. If β^* is raised from 3 mm to 1 cm, luminosity will be sacrificed by at most a factor of 2.3. In return, the structure of the IR becomes intrinsically simple. The proposed doublet scheme results in maximum betatron functions of only 30 km which represents a factor of 10 less than the 3 mm IR. At the same time, the natural chromaticities are about −500, which is more than a factor of 10 lower than a 3-mm IR. Also significant, local correction of chromaticity in the IR is eliminated. Chromaticity sextupoles can be placed in the arcs which correct the IR chromaticity, but contribute only minimally to the second-order tune spreads (as a function of amplitude). In constrast, an IR with high natural chromaticities requires local correction; often giving rise to large nonlinearities. In this case, correction sextupoles flanking the IP must be carefully placed and the lattice carefully designed so that nonlinear terms cancel to a high degree of accuracy. For example, the betatron functions at a pair of consecutive sextupole locations must be equal to at least 3-4 significant figures and their phase difference has to be exactly equal to π rad to 3-4 significant figures [7]. This level of cancellation is very difficult to maintain. With the IR designs presented in this paper, no high-degree of cancellation is required, the ring lattice is tunable, and, operationally, it is more stable.

Our main conclusion is that relaxing the low-beta criterion from 3 mm to at least 1 cm dramatically decreases both the length and complexity of the IR. It results in a collider ring design which is inherently stable and with a tuning range. In summary, it is our view that a small sacrifice of luminosity is worth the price of an operable machine.

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